

C. E. WILLIAMS

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COMMONWEALTH of AUSTRALIA
DEPARTMENT of CIVIL AVIATION

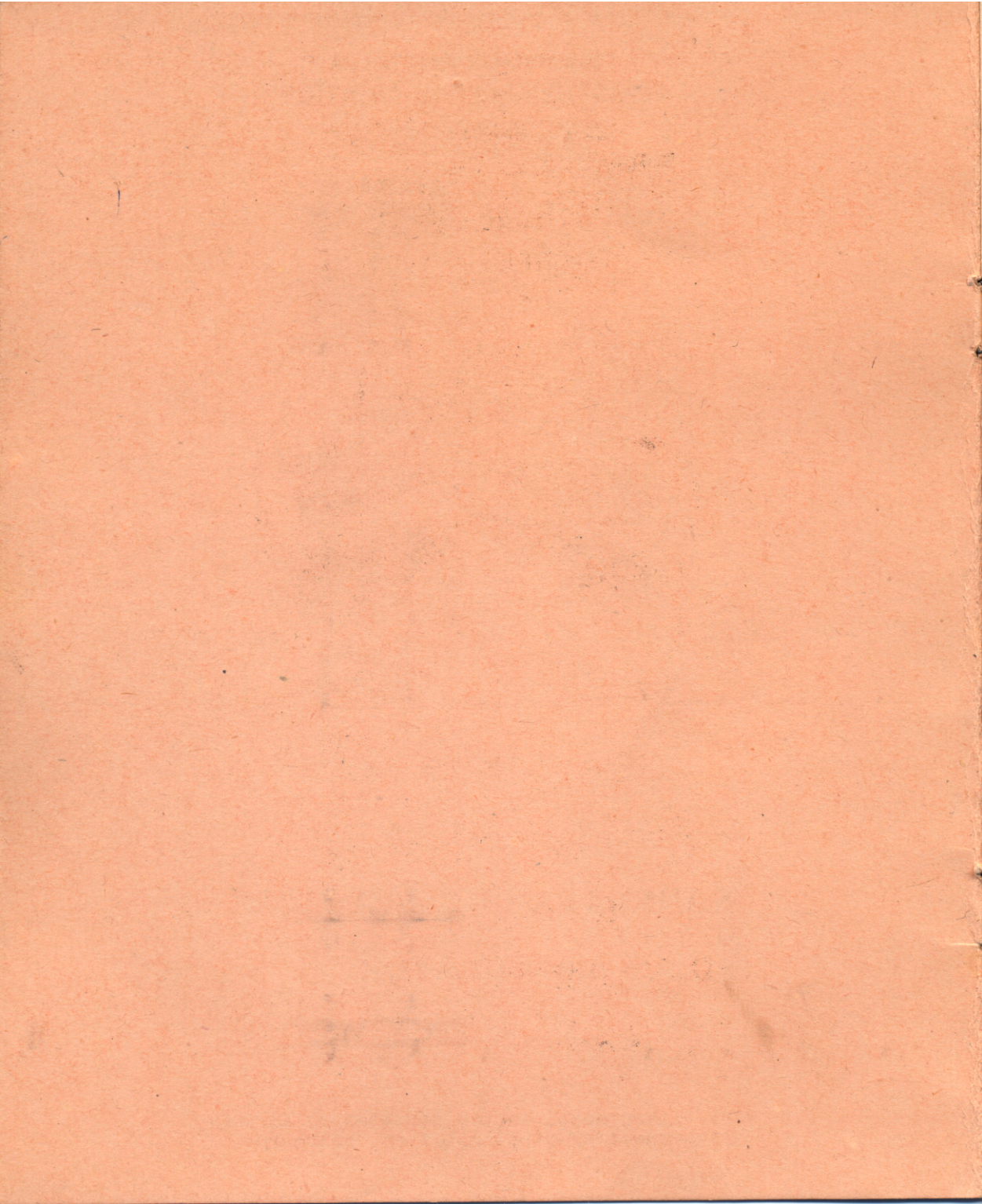


DIVISION of AIRWAYS



INSTRUCTION BOOK
for use with
RADIO TRANSMISSION LINE
CALCULATOR

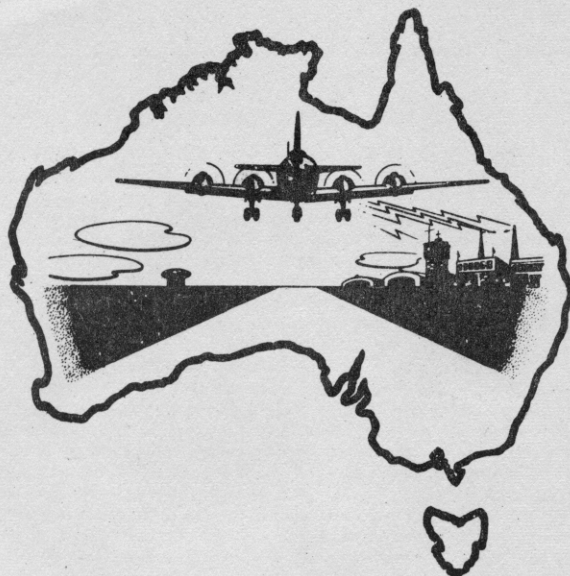
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*Photo-lithographed in the Division of Airways,
Department of Civil Aviation, Melbourne.*

An Improved TRANSMISSION LINE CALCULATOR

An extension of the "calculator" originally published in *ELECTRONICS* in January 1939.
New parameters have been added and accuracy has been improved

THERE was developed and has been in use in the Bell Telephone Laboratories for a number of years a particularly useful form of calculator for solving radio transmission line problems. The calculator was originally described in *ELECTRONICS** where it was presented in "cut-out" form. The impetus given to radio development by the war has promoted considerable interest in this calculator among engineers and research workers, particularly in the field of u-h-f technique where electrical measurements must be made indirectly. Accordingly, it has been felt desirable to again present at this time a comprehensive description of the device. Several new and useful parameters have been added to the original design and the entire calculator has been redrawn to improve its accuracy and facilitate reading the coordinates.

The calculator is, fundamentally, a special kind of impedance coordinate system, mechanically arranged with respect to a set of movable scales to portray the relationship of impedance at any point along a uniform open wire or coaxial transmission line to the impedance at any other point and to the several other electrical parameters. These other parameters are plotted as scales along the radial arm and around the rim of the calculator, both of which are arranged to be independently adjustable with respect to the main impedance coordinates. All of the para-

meters are related to one another and specific solutions to a given problem are obtainable through the use of an adjustable cross-hair index along the radial arm, which extends to intersect the scales around the rim. The parameters which are plotted on the calculator include:

I. Impedance, or admittance, at any point along the line. (a) Reflection coefficient magnitude. (b) Reflection coefficient angle in degrees.

II. Length of line between any two points in wavelengths.

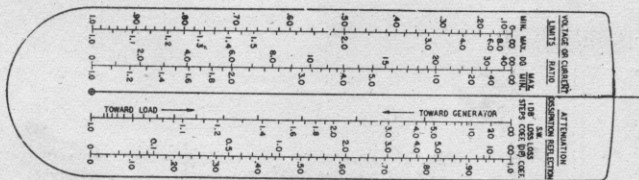
III. Attenuation between any two points in decibels. (a) Standing wave loss coefficient. (b) Reflection loss in decibels.

IV. Voltage or current standing

wise defined, normally considered to be that impedance which would be measured if the line were cut at that point and measurements were made looking into the line section which is connected to the load.

Impedance—General Considerations

The impedance at any point along the line and the power reaching this point from the generator completely determine the magnitude of the current and voltage and their phase relationship at that point. For a steady state, the generator impedance itself, as well as the impedance looking towards the generator from any point along the line where it may have been considered to have been cut,



wave ratio. (a) Standing wave ratio in decibels. (b) Limits of voltage and current due to standing waves.

A brief discussion of each of the several parameters and the manner in which they may be evaluated from the calculator will be given.

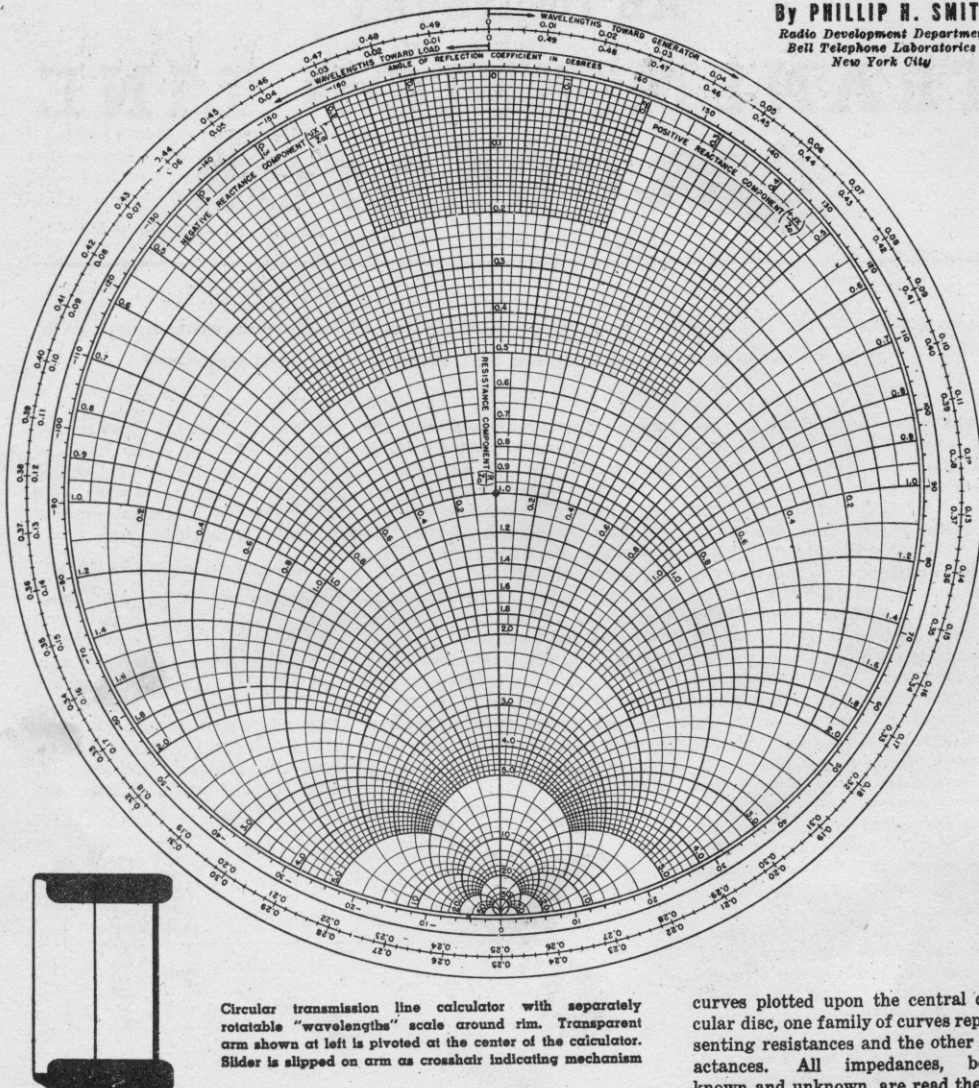
The impedance at any point along a transmission line is, unless other-

can in no way affect the *distribution* of current or voltage along the transmission line to which the generator is connected.

In other words, the generator impedance can have no effect upon the standing wave position or amplitude ratio or upon the relation of the standing wave to the impedance dis-

* Smith, Philip H. Transmission Line Calculator, *ELECTRONICS*, January 1939.

By PHILLIP H. SMITH
Radio Development Department
Bell Telephone Laboratories
New York City



Circular transmission line calculator with separately rotatable "wavelengths" scale around rim. Transparent arm shown at left is pivoted at the center of the calculator. Slider is slipped on arm as crosshair indicating mechanism.

tribution (locus of impedances) along the line. The generator impedance can affect *only* the power delivered to the transmission line and consequently the amplitude of the current or voltage all along the line, proportionately. The calculator relates the series components of the impedances thus considered at any point along a

transmission line to a number of other parameters which will be discussed individually.

Impedance Coordinates—Central Area of Calculator

The series impedance components are represented on the calculator as two orthogonal families of circular

curves plotted upon the central circular disc, one family of curves representing resistances and the other reactances. All impedances, both known and unknown, are read thereupon. To make the calculator of general application, these impedance coordinates are labeled as a fractional part of the characteristic impedance of the line (a fixed parameter in any given problem which may be evaluated from the physical dimensions of the line.)*

* Morrison, J. F., *Transmission Lines*, Pick-ups (Western Electric Co., New York), December 1939. See also "Radio Engineers Handbook," Terman, p. 174.

It is therefore necessary when using the calculator for solving problems involving the impedance at any point to first divide the components of all known impedances by the characteristic impedance of the line, and then obtain a solution from the calculator which, if an impedance, will be expressed as a fraction of the characteristic impedance of the line, and finally to multiply this solution by the characteristic impedance to obtain the answer in ohms. The characteristic impedance is usually a "real" number, i.e., a pure resistance, for radio transmission lines, which makes this procedure a relatively simple one.

The relation between the impedance of the line at any point and the other parameters listed above is evaluated with the aid of the cross-hair index line on the radial arm as described later.

Equivalent Parallel Components of Impedance

The calculator also provides a means for converting the series components of impedances to their equivalent parallel resistance and parallel reactance components. This is accomplished by setting the series components under the cross-hair index line on the radial arm and then moving the latter to the diametrically opposite point on the calculator and taking the reciprocal of the values read at that point as the equivalent parallel resistance and parallel reactance. (A reciprocal scale is provided along the radial arm.) The equivalent parallel component of resistance is useful in problems where it is desired to evaluate the magnitude of the voltage and to avoid converting the problem to one involving admittances.

Admittance Coordinates

The calculator relates the series components of admittances, as well as impedances, to the various other parameters listed above and accordingly the coordinates may be considered to be admittance coordinates if preferred. In this case the coordinate axis (real) labeled "Resistance Component (R/Z_0)" becomes the Conductance Component (a/Y_0) axis, the scale units then indicating a fractional part of the characteristic ad-

mittance of the line. Likewise, the coordinate axis (imaginary) labeled "Positive Reactance Component ($+jX/Z_0$)" becomes the Positive Susceptance Component ($+jb/Y_0$) axis and in the negative direction the Negative Susceptance Component ($-jb/Y_0$).

Admittance is defined as $Y = a + jb$ and it is important to remember that capacitance is considered to be a positive susceptance and inductance a negative susceptance. The direction of rotation indicated on the calculator in moving from one point to another is the same whether im-

terms of the magnitude and phase relationship of reflected and incident traveling waves, i.e., the reflection coefficient of the transmission line under consideration.

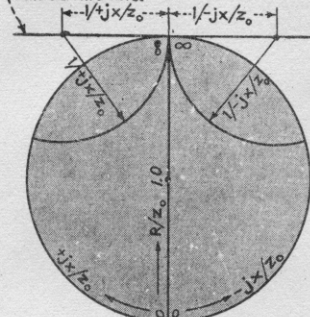
The magnitude of the reflection coefficient is expressed by a number between 0 and 1.0, which represents the ratio of reflected to incident voltage at the point under consideration. If the attenuation of the line is negligible the magnitude of the reflection coefficient will be a constant at all points along the line for a given load impedance resulting in a given standing wave amplitude ratio along the line.

The magnitude of the reflection coefficient is plotted as a scale along the radial arm. The phase angle of the reflection coefficient is directly related to the impedance and accordingly is indicated on the calculator as a scale around the rim of the impedance coordinate system.

All impedances radially in line have a constant reflection coefficient phase angle. This phase angle is the angle by which the reflected wave lags the incident wave at the point along the line under consideration. Where these two waves add in phase to give a maximum voltage the impedance is resistive and greater than the characteristic impedance of the line, and the angle of the reflection coefficient is zero degrees. Going towards the generator from this point, the departure from zero phase angle is linearly related to the distance traversed. The reflected voltage wave at first lags the incident voltage wave (having the longer path to traverse) and the phase angle of the reflection coefficient is considered to be negative for the first quarter wavelength from the voltage maximum point in the direction of the generator. The reactive component of the impedance in this region is negative.

At the exact quarter-wavelength (90 deg.) point the incident and reflected voltage waves are exactly out of phase and the angle of the reflection coefficient is ± 180 deg. Continuing in the same direction towards the generator, the two waves become increasingly more in-phase and in this region between one-quarter and one-half wavelength from the voltage maximum point towards

Construction line, tangent to circle at $R/Z_0 = \infty$. Note: Center points of all "Constant Reactance" circles fall on this line.



Construction of "constant reactance" curves $\pm jx/z_0$.

pedance or admittance coordinates are considered.

Converting Impedances to Admittances

Impedances may be converted to admittances, or vice versa, by going to a diametrically opposite point on the calculator, as described above for obtaining the equivalent parallel resistance components, and reading the values at that point as conductance and susceptance.

Reflection Coefficient

The impedance (or admittance) at any point along a uniform transmission line is completely defined by the amplitude and phase angle of the "reflection coefficient" at that point. It is often convenient to think of transmission line phenomena in

the generator, where the reactive component of the impedance is inductive, the reflected wave leads the incident wave and the reflection coefficient has a positive angle.

The relationship between the magnitude of the reflection coefficient and the standing wave amplitude ratio may be derived from the fact that at the voltage maximum point the incident and reflected waves add in phase, whereas, at the voltage minimum point they are exactly out of phase, thus

$$\frac{E_{max}}{E_{min}} = \frac{V_i + V_r}{V_i - V_r} = \frac{1 + \frac{V_r}{V_i}}{1 - \frac{V_r}{V_i}} \quad (1)$$

$$\text{Since } \frac{V_r}{V_i} = k,$$

$$\frac{E_{max}}{E_{min}} = \frac{1 + k}{1 - k} \quad (2)$$

k = magnitude of the voltage reflection coefficient

V_r = reflected voltage

V_i = incident voltage

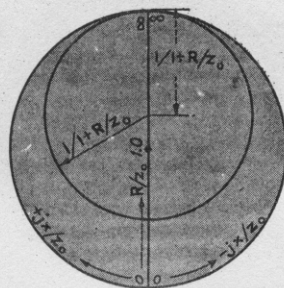
Length of Line—Movable Distance Scale Around Rim

Impedances along a uniform transmission line vary cyclically, repeating every half wavelength if the line has negligible attenuation. Thus, for any given termination the impedance locus path in going along the line in either direction from any initial starting point will close upon itself in exactly one-half wavelength effective length. The circular calculator is arranged so that one trip around the impedance coordinated disc at any constant distance from the center corresponds to a movement of just one-half wavelength along the transmission line. The length scale around the rim of the calculator is linear and its zero position may be adjusted so that measurements can be started from a point radially in line with any known impedance point on the coordinates and carried in either direction, i.e., either "towards the generator" or "towards the load" to a point where it may be desired to know the impedance.

Uniform transmission lines with air dielectric and negligible attenuation have an "effective length" equivalent to the length of the wave in free space and no correction is required for the length scale. However, any solid dielectric material in the field of the conductors causes a

reduction in the length of the standing wave which is proportional to $1/\sqrt{K}$ where K is the dielectric constant. This applies to lines where the entire field is confined to a homogeneous dielectric such as rubber-insulated coaxial lines. In coaxial lines, for example, where bead insulators are used, if the beads are spaced closer than about $1/36$ th wavelength, the line may be considered to have a uniform effective dielectric constant. The length scale refers to the "effective length" of the line.

As further discussed in the section entitled "Standing Wave Ratio," the relation between impedance and current distribution (standing waves), especially with respect to their position along the line, is often conveniently referred to the pure resistance



Construction of "constant resistance" curves R/Z_0 .

points and length measurements are often made with reference to one end or the other of the "real" axis, at which points the maximum and minimum current and voltage points occur.

Any line length in excess of one-half wavelength can be reduced to an equivalent shorter length to bring it within the scale range of the calculator by subtracting the largest possible whole numbers of half wavelengths therefrom.

Scales Along Radial Arm

A number of the parameters are uniquely related to one another as well as to the magnitude of the reflection coefficient previously described. These parameters are conveniently plotted as scales along the

radial arm in nomograph form. Their relationship may be evaluated through the use of the sliding index line which permits reading any or all of the several scales at the intersection of the slider index line. A given set of such values are also related to a given impedance locus which is traversed upon the impedance coordinates at the cross index line intersection when the radial arm is rotated once around the calculator. The several related parameters plotted along the radial arm in nomograph form include the following:

- (1) Attenuation in 1-db steps (due to loss resistance of line, leakage, and dielectric loss).
- (2) Standing wave loss coefficient (due to increased average current and voltage).
- (3) Reflection loss (or gain) in decibels.
- (4) Reflection coefficient magnitude (voltage).
- (5) Standing wave ratio (SWR) of maximum to minimum current or voltage.
- (6) Standing wave ratio expressed in decibels $(20 \log_{10} SWR)$.
- (7) Relative voltage or current at maximum and minimum points for constant power.

Attenuation

Attenuation causes the impedance locus along a uniform transmission line to spiral inward toward the center of the calculator from the initial starting point when going from the load end of the line "toward the generator", and to spiral outward toward the rim when going from an initial starting point "toward the load". The rate at which this spiral locus approaches the center (or the rim) depends directly upon the attenuation per unit length of line as well as upon the initial starting point.

Impedances near the rim (encountered along a line bearing a large standing wave) are altered to a greater extent by a db unit of attenuation, for example, than impedances near the center. The attenuation scale is conveniently plotted along the radial arm since it is a measure

of the rate at which the impedance locus spirals in or out.

The starting point for this non-linear scale must be capable of being set at any impedance point on the coordinates. Also, the scale must be capable of measuring attenuation in either direction from the starting point depending upon whether conditions are to be observed in a direction from an initial starting point "toward the load" or "toward the generator". To accomplish this the scale is laid out without markings, in 1-db steps. Thus, to take into account an attenuation of say 3 db it is necessary to count off three 1-db intervals in the proper direction along the attenuation scale from whatever starting point may have existed, before reading the answer on the impedance coordinates. The proper direction to go in correcting the impedance for attenuation of the line is indicated upon the attenuation scale itself.

Standing Wave Loss Coefficient

The scale along the radial arm labeled "S.W. Loss Coef." shows the additional copper or dielectric loss due to the presence of standing waves in the vicinity of the standing wave measurement. This added loss coefficient, which multiplies the calculated loss in decibels for a matched line, does not affect the line impedance. This added dissipation within the line is caused by the fact that the line conducts more average current and is required to withstand more average voltage for a given transmitted power when standing waves are present than would normally be the case if the line were properly matched.

Since copper losses at any point are proportional to the square of the current and dielectric losses or leakage losses are proportional to the square of the voltage, the percentage increment in losses applies equally to either type of loss. This loss coefficient refers more accurately to the increase in losses over a half wavelength of transmission line in the immediate vicinity of the standing wave measurement. In cases where the copper and dielectric or leakage losses are approximately equal it holds closely for any fractional part of a half wavelength. In this special

case, when moving along the line, the change in copper loss due to the standing current wave is approximately compensated by an equal and opposite change in dielectric or leakage due to the reversed slope of the voltage wave, resulting in a substantially uniform increase in loss for any fractional part of a half wavelength. If, due to attenuation, the standing wave ratio and consequently the standing wave loss coefficient change in moving along the line, for example several wavelengths, then the increased loss for the entire line section traversed lies between the coefficient limits indicated at each end.

Reflection Loss (or Gain)

The "reflection loss" may be derived from the reflection coefficient (k), which as described is the ratio of reflected to incident voltage. The reflection loss in decibels is

$$db = \log_{10} 10 \frac{P_{\text{absorbed}}}{P_{\text{incident}}} \quad (3)$$

The absorbed power is proportional to the square of the incident voltage (V_i^2) minus the square of the reflected voltage (V_r^2) and the incident power is proportional to the square of the incident voltage (V_i^2). Therefore

$$db = 10 \log_{10} \frac{V_i^2 - V_r^2}{V_i^2} \quad (4)$$

$$= 10 \log_{10} 1 - (V_r/V_i)^2 \quad (5)$$

$$= 10 \log_{10} (1 - k^2) \quad (6)$$

If the attenuation of the line is negligible, the "reflection loss" does not represent an actual loss of power, for if an impedance match is made at the sending end of the transmission line with the generator, a "reflection gain" takes place which neutralizes the "loss" at the load end. When the attenuation is not negligible, the reflection loss at the input (which actually represents an equivalent reflection gain when the input impedance to the line is matched to the generator impedance) will be less than the reflection loss at the load. This difference between the reflection loss, which can be read on the radial arm of the calculator, at the two ends of the line represents an additional dissipation loss within the line itself likewise, go through maximum and minimum values.

The measurement of standing waves is often accomplished through

the use of sliding capacitive or inductive probes (depending upon whether voltage or current waves respectively are to be observed). The output power taken from the probe is at a low level compared to that flowing in the main line so as not to disturb the line characteristics.

Standing Wave Ratio Expressed in Db

The probe output is amplified through a double detection receiver. The receiver includes an attenuator in its i-f amplifier circuits which is calibrated in decibels. The rectified output of the receiver is indicated on a reference level meter. It is convenient to adjust the meter output to an arbitrary reference mark and observe the change in attenuation required when going from a maximum to a minimum point along the standing wave. The standing wave amplitude ratio may then be expressed in decibels. Thus, a 6-db standing wave will have a ratio of maximum to minimum amplitude of 2 to 1. Used in this sense the term has no significance insofar as loss or power ratio is concerned. A scale is provided to permit expressing the standing wave ratio in decibels.

Relative Voltage or Current at Maximum and Minimum Points

If a transmission line is conducting a given amount of power it will do so most efficiently when standing waves are eliminated. However, there are cases when it will be acceptable or even desirable to permit standing waves to exist. In this case the line must be designed to withstand the increase in current and in voltage at the antinodes. This increase at the antinodes in both current and voltage (and decrease at the nodes) is plotted along the radial arm and refers to the increase or decrease at these points over what it would be if the line were properly terminated and were conducting the same amount of power to the load.

The voltage magnitude (E) at any point along the line in terms of the equivalent parallel-resistance (R_{par}) component and the power (P) is

$$E = \sqrt{R_{par} \times P} \quad (7)$$

whereas the current magnitude (I)

at any point in terms of the series resistance (R_{in}) component and the power (P) is

$$I = \sqrt{\frac{P}{R_{in}}} \quad (8)$$

In either case, the reactive component is not involved.

At the maximum and minimum impedance points the series and parallel components become the same. At these points the maximum and minimum current and voltage are conveniently evaluated from the standing wave ratio, characteristic impedance, and power as follows:

$$I_{max} = \sqrt{\frac{P}{SWR \times Z_0}} \quad (9)$$

$$I_{min} = \sqrt{\frac{SWR \times P}{Z_0}} \quad (10)$$

$$E_{max} = \sqrt{\frac{P \times Z_0}{SWR}} \quad (11)$$

$$E_{min} = \sqrt{P \times Z_0 \times SWR} \quad (12)$$

where Z_0 = characteristic impedance in ohms.

P = power in watts.

SWR = voltage or current standing wave ratio expressed as a number greater than unity.

Example for Use of the Calculator

A coaxial r-f transmission line having a characteristic impedance of $50 + j0$ ohms is terminated in an unknown impedance which causes a standing wave near the load such that $E_{max}/E_{min} = 2.0$. A voltage maximum point on the standing wave exists 0.175 wavelength from the load. The line is 2.84 wavelengths long and has 1.0 db attenuation.

A. To find the load impedance:

- (1) Set the slider on the radial arm to the position on the E_{max}/E_{min} scale opposite 2.0.
- (2) Rotate the radial arm until its index line coincides with the R/Z_0 axis between 1.0 and ∞ (where the voltage is maximum), and rotate the length scale around the rim until its 0 point is aligned with the index line on the radial arm.
- (3) Rotate the radial arm 0.175 wavelength counterclockwise "towards load" (from the voltage maximum point) as measured along the length scale at the rim and read the series components of the load impedance as $R/Z_0 = 0.60$ and $jX/Z_0 = 0.36$. Since Z_0 is $50 + j0$

ohms this corresponds to $Z_L = 50(0.60 + j0.36) = 30.0 + j18.0$ ohms.

B. To find the input impedance:

- (1) As in A(1) and A(2) above
- (2) Rotate the radial arm $2.84 - 2.50^* = 0.34$ wavelength clockwise "toward generator" as measured along the length scale and read the series input impedance components (before correcting for attenuation) as $R/Z_0 = 0.64$ and $jX/Z_0 = 0.44$. Since Z_0 is $50 + j0$ ohms, this corresponds to $Z_{in} = 50(0.64 + j0.44) = 32.0 + j22.0$ ohms.
- (3) Correct for attenuation by moving the slider index only along the arm "toward generator" 1 decibel unit. The input impedance components are then read as $R/Z_0 = 0.72$ and $jX/Z_0 = 0.37$, which corresponds to $Z_{in} = 50(0.72 + j0.37) = 36.0 + j18.5$ ohms.

C. To find the total dissipation in the line:

- (1) The attenuation as stated in the problem is 1 db, which is the nominal loss without standing waves. The increased attenuation due to standing waves (as observed at intersection of slider index with "S.W. Loss Coef." scale) is 1.25 times at the load end and 1.14 times at the generator end. The slider is set respectively as for B(2) and B(3) above. The dissipation loss for the whole line can be shown to be increased due to standing waves by the difference read on the reflection loss scale at the two ends of the line. This is seen to be $0.51 - 0.31 = 0.20$ db. Thus, in this case the total dissipation loss within the line is 1.20 db.

D. To find the increase in voltage or current at the maximum point due to standing waves:

- (1) With the slider index set as at B(2) and B(3) respectively, the increase over the uniformly distributed voltage (no standing waves)

due to the mismatch of impedance at the load is seen from the "Limits" scale to be 1.41 times at the load end and 1.31 times at the sending end of the line. At the nulls it is reduced to 0.707 and 0.761 times respectively. The actual magnitude of the voltage depends upon the power.

Construction Data

The data for construction of the main impedance coordinates of the calculator is shown in the figure.

It will be seen that all of the circles of constant resistance are centered on the resistance (R/Z_0) axis between the limits where $R/Z_0 = 1.0$ and ∞ and that these circles are all tangent to the edge of the coordinate system at the point when $R/Z_0 = \infty$.

The circles of constant reactance are all centered along a straight line perpendicular to the R/Z_0 axis at the point where $R/Z_0 = \infty$.

The scales along the radial arm of the calculator are conveniently plotted as a function of the magnitude of the voltage reflection coefficient k (a linear scale running between 0 at the center and 1.0 at the rim). The formulas utilized are:

Minimum Voltage or Current (N):

$$k = 1 - \frac{2}{1 + (1/N)^2} \quad (13)$$

Maximum Voltage or Current (X):

$$k = 1 - \frac{2}{1 + X^2} \quad (14)$$

Standing Wave Ratio (db):

$$k = 1 - \frac{2}{1 + \log^{-1} \text{db}/20} \quad (15)$$

Standing Wave Ratio, greater than unity (SWR):

$$k = 1 - \frac{2}{1 + SWR} \quad (16)$$

Attenuation (db):

$$k = 1 - \frac{2 \tanh(0.11512 \text{ db})}{\tanh(0.11512 \text{ db}) + 1} \quad (17)$$

Standing Wave Loss Coefficient (C):

$$k = 1 - \frac{2}{1 + C + \sqrt{C^2 - 1}} \quad (18)$$

Reflection Loss in db:

$$k = \sqrt{\frac{\log^{-1} \text{db}}{10} - 1} \quad (19)$$

* Subtract the largest whole number of half wavelengths to obtain equivalent length less than one-half wavelength.

Appendix 1 : NOTES ON MATCHING

It is desirable to operate an R. F. transmission line with a low standing wave ratio for a number of reasons:

1. The power handling capacity of the line as limited by the breakdown voltage of the insulation is higher.
2. The efficiency of the line is higher. The efficiency of a transmission line is defined as the ratio of the power output of the line (at the receiving end) to the power input (at the transmitting end). The efficiency depends on the attenuation and the standing wave ratio.
3. The input impedance (and hence the power input and output) is less sensitive to small changes of frequency or length of line.
4. There is less danger of pulling the frequency of the transmitter oscillator.

USE OF SMITH CHART FOR SOLUTION OF STUB-MATCHING PROBLEMS.

Preliminary Observations of Standing Waves.

1. The points of maximum and minimum voltage will be resistive.
2. The impedance at $V_{\max}$ will be higher than the line impedance R_0 .
3. The impedance at $V_{\min}$ will be lower than the line impedance R_0 .
4. From 2 and 3 therefore the impedance either side of the minimum may be calculated by considering 12 O'clock on the Smith Chart as being the voltage min. point.
5. Likewise 6 O'clock on the Smith Chart may be considered as a voltage max. point.

6. Since $\frac{R_0}{R}$ or $\frac{R}{R_0}$ = S.W.R.

then a circle of radius defined by R/R_0 or Z_{load}/R_0

will be identical to the circle defined by the S.W.R.

STUB MATCHING.

IMPEDANCE (Z) is defined as the ratio of voltage to current. It is a complex quantity. The real part is known as the resistance (R) and the imaginary part as the reactance (X). Inductive reactance is positive, capacitive reactance is negative.

ADMITTANCE (Y) is defined as the ratio of current to voltage, hence it is the reciprocal of impedance. The real part of admittance is known as conductance (G), and the imaginary part is known as susceptance (B). Inductive susceptance like inductive reactance is positive; capacitive susceptance like capacitive reactance, is negative.

Greatest convenience in working stub-matching problems is obtained by working in conductances, susceptances and admittances since line and stub are connected in parallel.

To eliminate standing waves on the line, the stub must be placed at a point on the line where the conductance is equal to characteristic conductance of the line, and the length of the stub must be adjusted to make the total susceptance at the point of attachment equal to zero.

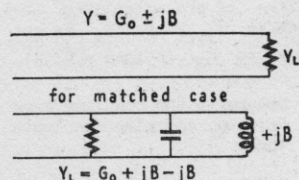
EXPLANATORY NOTE:

$$\text{Conductance } G = \frac{1}{R}$$

$$\text{Susceptance } B = \frac{1}{X}$$

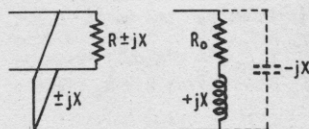
$$\text{Admittance } Y = \frac{1}{Z}$$

After conversion to admittances etc. the load becomes:



Avoid the fallacy of looking along line and putting stub of opposite sign and equal reactance at point found by:-

$Z = R_0 (1 \pm jX)$. Equiv. circuit is



from above it can be seen that this solution does not remove the reactance.

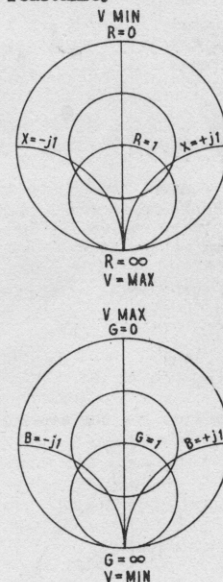


FIG. 1

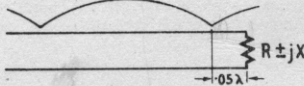
To convert from impedance to equivalent admittance draw a straight line through from the impedance point through the centre of the chart to intersect with the S. W. R. circle.

PROBLEM:

Reactive load to be matched to a line of characteristic impedance R_0 where R_L is not equal to R_0 ($Z_{\text{load}} = R_L \pm jX$).

1. Determine the Standing Wave Pattern and plot on Smith Chart.

S.W.R. = 3 and E_{min} occurs $.05\lambda$ from load



- (a) Plotting on Smith Chart S.W.R. = 3, load $.05\lambda$ from V_{min} (12 O'clock) 'Towards Load' gives $Z = R_0$ ($.36 - j.29$).

- (b) Convert load into terms of conductance and susceptance by rotating 180° .

i.e. Convert from $Z = R \pm jX$ to $Y_L = G \pm jB$. This gives admittance = G_0 ($1.7 + j1.31$) at load.

- (c) Search for two points along line having $Y = G_0$ ($1 \pm jB$)

Where conductance equals characteristic conductance plus a susceptance which is to be removed by the stub.

These points will be::

1. $Y = G_0 (1 + j1.15)$
and
2. $Y = G_0 (1 - j1.15)$

The position for the stubs has now been located.

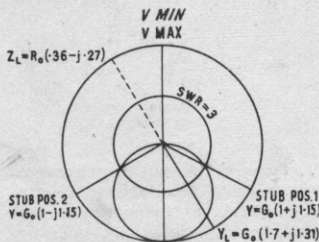


FIG. 2

Stub positions are located as follows:-

Distance from V_{min} (6 O'clock)

is for

1. (Wave lengths toward generator) $.25\lambda + .166\lambda = .416\lambda$

$\therefore$ Distance from load = $.416\lambda + .05\lambda = .466\lambda$

2. (Wave lengths toward generator) $.332\lambda - .25\lambda = .082\lambda$

$\therefore$ Distance from load = $.082\lambda + .05\lambda = .132\lambda$

- (d) Now calculate a stub having input susceptance $+j1.15$ for the solution nearest the load.

A second stub can be calculated having input susceptance $-j1.15$ for the position furthestmost from the load.

The length of the stub is chosen to present a susceptance B equal in magnitude but opposite in sign to the susceptance of the line at the point of attachment. The total susceptance at the stub is then zero, and the line is then terminated in its characteristic conductance at the stub.

- (e) Since a stub is more easily recognisable when expressed in terms of reactance, and having decided upon the susceptance required to cancel the existing susceptance, convert to reactance by plotting 'B' on the outer circumference of the Smith Chart (assuming that the stub has no resistance) and rotating 180° to find 'X'.

For an S/C stub calculate from 12 O'clock (V_{min}).
For an O/C stub calculate from 6 O'clock (V_{max}).
Method shown in Fig. 3.

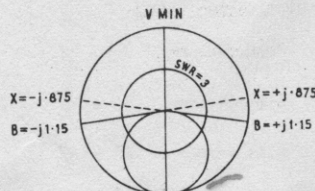


FIG. 3

STUB LENGTHS.

1. O/C stub (L_1) from V_{max} (towards gen) to $-j.875 = .136\lambda$
2. O/C stub (L_2) from V_{max} (towards gen) to $+j.875 = .25\lambda + .114\lambda = .364\lambda$
3. S/C stub (L_3) from V_{min} (towards gen) to $+j.875 = .114\lambda$
4. S/C stub (L_4) from V_{min} (towards gen) to $-j.875 = .25\lambda + .136\lambda = .386\lambda$

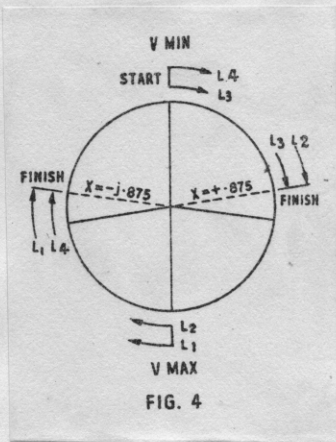


FIG. 4

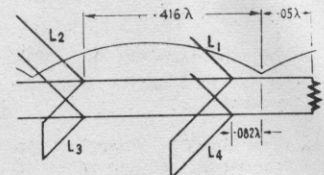
SOLUTION

FIG. 5

$$\begin{aligned} (L_1 &= L_4 - \frac{\lambda}{4} & L_1 &= .136\lambda \\ L_2 &= .364\lambda \\ L_3 &= L_2 - \frac{\lambda}{4} & L_3 &= .114\lambda \\ L_4 &= .386\lambda \end{aligned}$$

